# Do Non-Exporters Lose From Lower Trade Costs?*

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**Abstract**

We challenge the idea that trade liberalizations are detrimental to non-exporters: if they expect to export in the future, larger export profits increase their present value. To do this, we develop a model of international trade, where firm productivity follows a Geometric Brownian Motion, with a drift endogenously determined by innovation. Firms export when reaching a productivity threshold, after which they grow at a constant average rate, generating a firm distribution with Pareto upper tail. Non-exporters grow at an increasing rate, lower than that of exporters. We calibrate the model to US data. The anticipation of future export profits accounts for up to 10% of the value of non-exporters. Reducing trade costs increases export profits and reduces domestic ones. As a result, small non-exporters lose value driven by domestic profits, but larger ones gain because of the anticipation of future exports. This is consistent with empirical studies that find some non-exporters expand after liberalizations. A 1% reduction in trade costs reduces the value of non-exporters by 0.01% on average, but 59% of them actually gain, up to 0.13%, which is more than some exporters.

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# 1 Introduction

The export market is extremely dynamic: many of the firms that do not export today are very likely to start exporting soon. In spite of this, most research on international trade argues that lower trade barriers benefit exporters at the expense of non-exporters. But if the value of a non-exporter includes the present value of future exports, it follows that trade liberalizations may benefit non-exporters too, depending on how likely they are to export. This paper studies how trade barriers affect non-exporters when markets are as dynamic as the data suggests.

Our main results are that, indeed, trade liberalizations may benefit non-exporters. Some non-exporters even gain more than some exporters. Broadly speaking, most non-exporters gain, and only very unlikely exporters lose when trade barriers drop. We also find that adding the dynamic element to the model helps understand strong relationships between trade costs and firm characteristics, that are not consistent with more mainstream models.

First, our model accounts for a growth spurt in firms that are about to export, since the anticipation of export profits provides a boost to growth. Bernard and Jensen (1999) interpret the empirical finding as evidence that exporting is a consequence of an exogenous event leading firms to grow faster, which, being large, find it profitable to export. While we find this is possible, it is more likely that the reason for this growth spurt is future exports.

Second, we find that non-exporters on average increase their innovation when trade costs drop, as Bustos (2011) finds in Argentina, Eslava et al. (2015) in Colombia, Topalova and Khandelwal (2011) in India and Newman et al. (2013) in Vietnam. These findings appear puzzling at first, and have driven Newman et al. (2013) and Eslava et al. (2014) to develop models with additional channels to account for them. In our case this arises naturally, due to the anticipation of export profits that increase when trade costs fall.

Our model builds on Atkeson and Burstein (2010) and Costantini and Melitz (2008), both models with endogenous innovation. We differ however, in the innovation process itself, using a more general and realistic process. Atkeson and Burstein (2010) limit next period's productivity to be one of two alternatives, and Costantini and Melitz (2008) assume innovation is a once and for all type of investment. We write our model in continuous time, and firm productivity follows a Geometric Brownian Motion with a drift that is endogenously determined by innovation. Thus, at every instant, firms determine how many resources to devote to innovation, and the actual productivity outcome is a draw of a convex and random distribution, determined by innovation expenditure. The advantages of using a continuous time will be clear below. Not only are there gains in tractability, but also the results follow in a straightforward manner.

Some features of the equilibrium look like other existing models of international trade: large, high productivity firms export, while small, unproductive firms sell only domestically. Extremely unproductive firms exit. Other features are not so standard, but consistent with the empirical literature. Exporters grow faster than non-exporters. Bernard and Jensen (1999) find this to be true for shipments, employment, and labor productivity. The average growth rate of exporters is independent of size, as found by Berthou and Vicard (2013), accounting for Gibrat's law.

The average growth rate of non-exporters is increasing in productivity. This is partly because the option to export in the future increases the value of non-exporters, and has a larger effect on firms that are close to exporting (i.e., more productive ones). As a result, growth rates increase with productivity. It is as if there were increasing returns to scale when in fact there are none: as a non-exporter grows, it gains from an increase in their productivity and from an increase in the probability of becoming an exporter. For the largest non-exporters, the export option adds 13% to the growth rates, up from 8% for the median non-exporter. The export option also affects unlikely exporters, that would exit absent the export option.

In more technical terms, the value of non-exporters approaches that of exporters continuously, because the probability of exporting grows continuously with productivity. This also implies that the growth rate of non-exporters approaches continuously that of exporters.[1] This firm behavior gives shape to the aggregate distribution of firms, generating a Pareto upper tail, as in Gabaix

[1] These characteristics are implied by the value matching and smooth pasting conditions.

(2011), since the growth rate of exporters is constant, and exporters are the relatively larger firms.[2]

We use our framework to study the effects of a reduction in marginal (iceberg) trade costs. This reduction increases the value of firms that export. We prove this algebraically under special assumptions. These special assumption put bounds on the exogenous distribution from which new firms draw their productivity, and the size of a fixed cost require to produce. To show the effects in a more general case, we calibrate our model to the United States. Our numerical results confirm these findings. A 1% reduction in trade costs increases the average value of exporters by 0.133%. This includes both old exporters, and new ones. Old exporters gain 0.126% on average, and new exporters 0.142%.

In the case of non-exporters, there are two opposing effects that determine their reaction. On one hand, the flow profits of non-exporters drop, reducing growth. On the other, the value of the export option increases, increasing growth. We show algebraically that the value and growth rate of the largest non-exporters increase. These firms discount future exports at a small rate, so an increase in the value of an exporter increases their own. In the calibrated model, while the value of non-exporters drops on average by 0.011% per percentage point reduction in trade costs, 59% of non-exporters increase their value, and the increase can be of up to 0.132% per percentage drop in trade costs, which is more than what old exporters gain.

A similar result holds when looking at the ratio of innovation to sales, except that now the average ratio among non-exporters increases by 0.140% per percentage drop in trade costs. This is a novel result, and fits well into the findings of the empirical literature. Eslava et al. (2015) find that trade liberalization in Colombia increases the skill intensity in firms that are purely domestic: that is, they neither export nor import. Bustos (2011) finds that firms in all quartiles of the distribution increase innovation spending when trade costs fall, many of which are non-exporters. This is puzzling in the light of her model, since firms in the first quartile are not likely to be exporters. She rationalizes the finding by acknowledging that size is not a perfect proxy

[2] We derive it using the Kolmogorov Forward Equation.

for productivity. In the light of our model, her findings are perfectly consistent with these firms having relatively low productivities and not exporting. Topalova and Khandelwal (2011) find that lower trade barriers increase the productivity of all domestic firms in India, including small ones, unlikely to be exporters. Yan-Aw et al. (2011) find that a reduction in export costs increases the R&D of both exporters and non-exporters by performing counterfactuals on a model that is structurally estimated for Korean data. Newman et al. (2013) find that trade liberalizations increase the productivity of domestic firms in Vietnam, including non-exporters. They suggest that input output linkages make this possible. Eslava et al. (2014) build a model with input output linkages that can successfully account for the increase in innovation and productivity among non-exporters. This paper suggests that these input output linkages, though highly likely, are not necessary to rationalize their findings.

We are related to a number of papers that model endogenously the growth rate of firms in continuous time. These include Acemoglu and Cao (2010) and Luttmer (2007, 2010). These papers find that all firms grow at an average constant rate, consistent with Gibrat's law. While this is true for our exporters, non-exporters grow at a lower rate, consistent with several studies of firm behavior (see Bernard et al., 2007, for a survey), and this rate increases with productivity.

We are also related to Impullitti et al. (2013), who develop a continuous time version of Melitz (2003). With respect to them, we add an innovation decision. Our solution method shares many characteristics with them, that include smooth pasting and value matching conditions, and the use of a Kolmogorov Forward Equation to identify the aggregate firm distribution.

The evidence on the relation between trade and innovation is abundant. Bustos (2011) finds evidence of trade liberalization triggering innovation among firms in Argentina. Lileeva and Trefler (2010) find similar evidence among Canadian firms that expanded following the trade liberalization with the United States increased their innovation levels. Alvarez and Lopez (2005) find that exporting raises research and development expenditures. Caliendo and Rossi-Hansberg (2012) build a model that suggest a micro-founded model of why lower trade costs would drive innovation and growth, and Caliendo et al. (2015) find empirical support for their channel.

Our paper is organized as follows. Section 2 introduces our model of international trade and innovation. Section 3 explores the effects of a change in trade costs in a limited case where algebraic proofs are possible. To explore the effects in a more general case, section 4 calibrates the model to salient features in the United States. Section 5 shows features of the calibrated equilibrium. Section 6 presents our findings, and section 7 concludes.

# 2 The Model

Time is continuous. There are 2 symmetric countries that produce a continuum of tradable differentiated goods. Symmetry allows us to omit country sub-indices. A " * " denotes the foreign country.

**Preferences.** An infinitively lived representative consumer has the following preferences

$$U\left(\{q(\omega,t)\}\right) = \int_0^\infty e^{-\rho t} \ln Q(t) dt,$$
$$Q(t) = \left[\int_{\Omega(t)} q(\omega,t)^{\frac{\nu-1}{\nu}} d\omega + \int_{\Omega^*(t)} q(\omega,t)^{\frac{\nu-1}{\nu}} d\omega\right]^{\frac{\nu}{\nu-1}} \tag{1}$$

where $\omega$ is the name of the good, $\Omega(t)$ and $\Omega^*(t)$ are the sets of goods produced in the domestic and foreign countries at time $t$ respectively and $q(\omega,t)$ denotes consumption of good $\omega$ at time $t$. The elasticity of substitution between goods is $\nu > 1$ and the discount factor is $\rho > 0$.

**Technologies.** Incumbent firms make production, innovation, and exporting decisions. They are owned by domestic consumers, who receive lump-sum profits. Each firm is a monopolist. Given a productivity level $z$ and labor services $n$, we follow Atkeson and Burstein (2010) and assume that the firm producing good $\omega$ has technology:

$$y(\omega; z, n) = z(\omega)^{\frac{1}{\nu-1}} n$$

Productivity $z$ follows a Geometric Brownian Motion, given by

$$\dot{z}(t)dt = g(t)z(t)dt + z(t)\sigma dW(t) \tag{2}$$

where $g(t)$ is the average rate of growth of $z(t)$ and it is determined endogenously, $W(t)$ follows a Wiener process and $\sigma \geq 0$ is the standard deviation of the growth rate of $z(t)$.

A firm can innovate to increase its productivity level $z$. Setting the drift of the Geometric Brownian Motion process described in (2) equal to $g$ costs, in labor units,

$$c(z, g) = \frac{\kappa_I z}{2} g^2$$

where $\kappa_I > 0$ determines the innovation cost. Notice that we choose to elevate $g$ to the power 2. In theory, any convex function should work. Our choice is helpful to characterize the solution, delivering closed form solutions in many cases.

There is a flow production cost equal to $\kappa_P > 0$ units of labor. Exporting requires flow cost of $\kappa_X > 0$ units of labor. Exports are subject to iceberg trade costs. To export quantity $q$ an exporter must ship $\tau q$, where $\tau \geq 1$, and $\tau - 1$ is lost in transit.

**Firm Birth.** There is a large pool of potential entrants. To enter, a firm must spend $\kappa_E > 0$ units of labor. After entry, the new firm starts producing with a parameter $z$ drawn from a cumulative distribution function $G(z)$.

**Firm Death.** Death can be an endogenous decision or an exogenous shock. Any firm can die at any instant with a probability $\delta \in [0, 1]$. Similarly, a firm may choose to exit the market. The reasons why we impose these two risks of death are that $(i)$ small firms tend to exit the market more often than large firms, $(ii)$ some large firms also exit the market from time to time, and $(iii)$ exogenous death guarantees the existence of a stationary distribution of firms in steady state.

**Labor Market Clearing.** Let $M(t)$ be the measure of entrants in instant $t$. Let the measure of workers be $L$, then labor market clearing is

$$L = \int_{\Omega(t)} \left[ n(\omega,t) + c(z(\omega,t), g(\omega,t)) + \kappa_X \mathcal{I}(\omega,t) + \kappa_P \right] d\omega + M(t)\kappa_E \tag{3}$$

where

$$\mathcal{I}(\omega,t) = \begin{cases} 1 \text{ if firm } \omega \text{ exports at instant } t \\ 0 \text{ otherwise} \end{cases}$$

The first term inside the integral on the right hand side of equation (3) is labor used in production, the second is labor used in innovation, the third is labor used to cover fixed export costs, and the fourth covers fixed production costs. The last term is labor used to create new firms.

## 2.1 Steady State Equilibrium

We solve the model in steady state, and therefore drop the argument $t$. We normalize the wage rate, $w = 1$, so that all prices are in units of labor. Let $p(\omega)$ be the price of good $\omega$. In equilibrium a producer charges the same price independently of destination, so we do not introduce additional notation. Consumers solve

$$\begin{aligned} & \max \ln Q \\ s.t. \quad & Q = \left[ \int_{\Omega} q(\omega)^{\frac{\nu-1}{\nu}} d\omega + \int_{\Omega^*} q(\omega)^{\frac{\nu-1}{\nu}} d\omega \right]^{\frac{\nu}{\nu-1}} \\ & \int_{\Omega} p(\omega) q(\omega) d\omega + \tau \int_{\Omega^*} p(\omega) q(\omega) d\omega = 1 + \int_{\Omega} \pi(\omega) d\omega \end{aligned}$$

The last line is the budget constraint. $\pi(\omega)$ are profits of a firm $\omega$. Let the right-hand side be equal to $I$ (for income). The demand of good $\omega \in \Omega$ is

$$q(\omega; p, P, I, \tau) = \begin{cases} p^{-\nu} P^{\nu-1} I \text{ if } \omega \in \Omega \\ (\tau p)^{-\nu} P^{\nu-1} I \text{ if } \omega \in \Omega^* \end{cases} \tag{4}$$

where $P$ is the Dixit-Stiglitz aggregate price:

$$P = \left[ \int_{\Omega} p(\omega)^{1-\nu} d\omega + \tau^{1-\nu} \int_{\Omega^*} \mathcal{I}(\omega) p(\omega)^{1-\nu} d\omega \right]^{\frac{1}{1-\nu}} \tag{5}$$

The problem of the firm can be split into a static decision and a dynamic decision. The first involves how much to produce and the price given their current productivity, and the second involves how much to innovate, whether to export, and whether to produce or not. The static decision results from solving:

$$\begin{aligned} &\max_{p, p_x, q_n, q_x, n} \quad p_n q_n + \tau p_x q_x - n \\ s.t. \quad & q_n + q_x = z(\omega)^{\frac{1}{\nu-1}} n, \quad q_n = p_n^{-\nu} P^{\nu-1} I, q_x = (\tau p_x)^{-\nu} P^{\nu-1} I \end{aligned}$$

The solution to this problem is the mark-up rule $p(\omega) = p_n(\omega) = p_x(\omega) = \frac{\nu}{\nu-1} z(\omega)^{\frac{-1}{\nu-1}}$. Let $\hat{\pi}_n(P, I, z)$ be the variable profits for a non-exporter (profits before paying innovation or fixed export or production costs) and $\hat{\pi}_x(P, I, z, \tau)$ be the same for exporters.

$$\hat{\pi}_n(z(\omega), P, I) = \nu^{-1} I P^{\nu-1} z(\omega) = \pi_n z(\omega) \tag{6}$$

$$\hat{\pi}_x(z(\omega), P, I, \tau) = \hat{\pi}_n(z(\omega), P, I) + \tau^{1-\nu} \hat{\pi}_d(z(\omega), P^*, I^*) = \pi_x z(\omega) \tag{7}$$

As in Dixit and Stiglitz (1977), the name of the good $\omega$ is irrelevant for decisions, only the productivity level $z$ matters. Accordingly, in what follows we drop out the name of the good and make all functions dependent only on $z$.

Dynamic decisions include whether to produce, whether to export, and how much to innovate. The value function is

$$V(z) = \max\{0, V_n(z), V_x(z)\} \tag{8}$$

The value of not producing is 0. The value of a firm that does not export is $V_n(z)$, and the value of an exporter is $V_x(z)$. For non-exporters, the Hamilton-Jacobi-Bellman equation is

$$(\rho + \delta)V_n(z) = \max_g \left\{ \pi_n z - \kappa_P - \frac{\kappa_I z}{2} g^2 + gzV_n'(z) + \frac{1}{2}\sigma^2 z^2 V_n''(z) \right\} \tag{9}$$

where we have assumed that the probability that the firm will become an exporter in a small period $dt$ is equal to zero. Similarly, since for a sufficiently small $dt$ the probability of losing the exporting status is zero, $V_x(z)$ solves:

$$(\rho + \delta)V_x(z) = \max_g \left\{ \pi_x z - \kappa_P - \kappa_X - \frac{\kappa_I z}{2} g^2 + gzV_x'(z) + \frac{1}{2}\sigma^2 z^2 V_x''(z) \right\} \tag{10}$$

The free entry condition guarantees that the value of a firm when entering the market must equal the cost to enter. Recall that $G(z)$ is the exogenous distribution of entrant productivities. The free entry condition is

$$\int_{-\infty}^{+\infty} V(z)G(z)dz = \kappa_E \tag{11}$$

Because the productivity of firms is stochastic, firm sizes are constantly changing, driven by an endogenous drift and an exogenous volatility, so the distribution of firms in equilibrium is endogenous. Call this distribution $\mu(z,t)$. This must satisfy the Kolmogorov Forward Equation, which implies:

$$\frac{\partial\mu(z,t)}{\partial t} = -\,\mu(z,t)\left(\frac{\partial g(z,t)}{\partial z}z + g(z,t) + \delta - \sigma^2\right) - \\ \frac{\partial\mu(z,t)}{\partial z}z(g(z,t) - 2\sigma^2) + \frac{\partial^2\mu(z,t)}{\partial z^2}\frac{\sigma^2 z^2}{2} + MG(z)$$

In steady state, $\frac{\partial\mu(z,t)}{\partial t} = 0$. Dropping the argument $t$, the second order differential equation that determines the firm size distribution is

$$0 = \mu(z)\left(g'(z)z + g(z) + \delta - \sigma^2\right) + \mu'(z)z(g(z) - 2\sigma^2) - \frac{\sigma^2 z^2}{2}\mu''(z) - MG(z) \tag{12}$$

## 2.2 Growth rates

The growth rates of exporters potentially depend on $z$. We require that the $\lim_{z\to\infty} g(z)$ is bounded for the problem of the firm to be well defined. Also, we require $g(z) \geq 0$ for all $z$. These observations lead us to look for an equilibrium where the growth rate of exporters converges to a positive constant. A key result of our analysis is that this does not always exists. In particular, we show in Appendix A that a necessary condition for existence is that, in equilibrium, $2\pi_x < (\rho+\delta)^2$. This depends on the value of parameters, since $\pi_x$ is an equilibrium variable. We assume this holds, and then verify that it does in equilibrium. We limit our analysis to cases in which the inequality holds.[3]

We next show how growth rates look like in equilibrium. The Bellman equations are described in equations (9) and (10). Notice that we can normalize everything with respect to the innovation

[3]This is also present in Atkeson and Burstein (2010), see their footnote 8. If the condition does not hold, it is possible that an equilibrium with oscillating growth rates for large exporters exists. We do not consider this possibility on the basis that it is counterfactual.

cost $\kappa_I$ by writing the Bellman equation as

$$(\rho+\delta)\hat{V}_n(z) = \max_g \hat{\pi}_n z - \hat{\kappa}_P + \frac{g^2 z}{2} + \hat{V}_n'(z) g z + \frac{\hat{V}_n''(z)(z\sigma)^2}{2},$$
$$(\rho+\delta)\hat{V}_x(z) = \max_g \hat{\pi}_x z - \hat{\kappa}_P - \hat{\kappa}_X + \frac{g^2 z}{2} + \hat{V}_x'(z) g z + \frac{\hat{V}_x''(z)(z\sigma)^2}{2}$$

where the symbol $\hat{}$ implies that the function or parameter is divided by $\kappa_I$. For example, $\hat{\pi}_n = \frac{\pi_n}{\kappa_I}$ is profits relative to the cost of innovation. Since this normalization is a monotonic transformation on the original problem, it has no effect on the choice of $g$, the growth rate, which also implies that this has no effect on the distribution of firms in equilibrium. Also, as long as we normalize the entry cost relative to the innovation cost, this normalization does not change the free entry condition either. Thus, all costs can be normalized relative to the innovation cost, which reduces the number of parameters. It is convenient to drop the $\hat{}$, understanding that all functions and variables are relative to $\kappa_I$.

The equilibrium is characterized by productivity thresholds $z_e$ and $z_x$, such that firms with productivity $z < z_e$ exit the market, firms with productivity $z \in [z_e, z_x]$ sell only domestically, and firms with $z > z_x$ sell domestically and export. Firm growth rate also depends on these thresholds. Using the first order conditions with respect to $g$ in each interval,

$$g(z) = \begin{cases} V_n'(z) & \text{if } z \in [z_e, z_x] \\ V_x'(z) & \text{if } z > z_x \end{cases}$$

Introducing the solution in the Bellman equation,

$$(\rho+\delta)V_n(z) = \left(\pi_n + \frac{1}{2}V_n'(z)^2\right) z + \frac{1}{2}\sigma^2 z^2 V_n''(z) - \kappa_P, \quad \forall z \in [z_e, z_x] \tag{13}$$
$$(\rho+\delta)V_x(z) = \left(\pi_x + \frac{1}{2}V_x'(z)^2\right) z + \frac{1}{2}\sigma^2 z^2 V_x''(z) - \kappa_X - \kappa_P, \quad \forall z > z_x \tag{14}$$

### 2.2.1 Exporters

Start with exporters. The ordinary differential equation that defines their problem is

$$(\rho+\delta)V_x(z)=\pi_x z-\kappa_X-\kappa_P+\frac{1}{2}zV_x'(z)^2+\frac{1}{2}(\sigma z)^2V_x''(z)^2 \tag{15}$$

for $z\in(z_x,\infty)$. This is a second order ordinary differential equation, and thus requires two boundary conditions for its solution. The first boundary condition assumes that, as $z\to\infty$, the value of the firm approaches the value of a firm that never faces the risk of exit or stop exporting. Thus,

$$\lim_{z\to\infty}V_x(z)=\lim_{z\to\infty}\left[\frac{\pi_x z-\frac{1}{2}zg_x(z)^2-\kappa_X-\kappa_P}{\rho+\delta-g_x(z)}\right] \tag{16}$$

Using equations (15) and (16), it follows that, as $z\to\infty$, there are two growth rates that solve the equations:

$$\begin{aligned} g_{x1}&=(\rho+\delta)\left(1-\sqrt{1-\frac{2\pi_x}{(\rho+\delta)^2}}\right)\\ g_{x2}&=(\rho+\delta)\left(1+\sqrt{1-\frac{2\pi_x}{(\rho+\delta)^2}}\right)\end{aligned} \tag{17}$$

The following proposition shows that if the growth rate were $g_{x2}$, firms would make flow losses as $z\to\infty$. Hence, only $g_{x1}$ is a profit maximizing solution.

**Proposition 1** *If* $\lim_{z\to\infty}g(z)=g_{x2}$, *then firms would make flow losses when they become too large.*

**Proof:** Flow profits are $\pi_x z-\frac{1}{2}zg_{x2}^2-\kappa_P-\kappa_X$. To show that these are negative, it is enough to show that $\pi_x - \frac{1}{2}g_{x2}^2$ is a negative number.

$$\pi_x - \frac{1}{2}g_{x2}^2 = \pi_x - \frac{1}{2}(\rho+\delta)^2\left(1+\sqrt{1-\frac{2\pi_x}{(\rho+\delta)^2}}\right)^2$$
$$= 2\pi_x - (\rho+\delta)^2 - \frac{1}{2}(\rho+\delta)^2\left(2\sqrt{1-\frac{2\pi_x}{(\rho+\delta)^2}}\right) < 0$$

The last inequality follows from the fact that $2\pi_x - (\rho+\delta)^2 < 0$. Otherwise, the solution would be imaginary. ■

Thus, the solution implies that $g_x(z) \to g_{x1}$ as $z \to \infty$. Notice that in the limit, the growth rate is constant, so that $V_x'(z)$ is constant, and the value function of non-exporters becomes linear.

To identify the second condition, we force the model to deliver Gibrat's law: the growth rate of large firms is independent of firm size (see Luttmer, 2007). Berthou and Vicard (2013) confirm this relationship for all exporters in France. In the model, this implies that $g_x(z) = g_{x1}$ for all $z$. Moreover, since $g_x(z) = V_x'(z)$, this implies $V_x''(z) = 0$ for all $z$, so that equation 15 becomes a first order differential equation, and only one border condition is necessary. The resulting value function of exporters is:

$$(\rho+\delta)V_x(z) = \pi_x z - \kappa_X - \kappa_P + \frac{1}{2}z g_{x1}^2 \tag{18}$$

Notice that the constant average growth rate among exporters is different than what Burstein and Melitz (2011) find, since they find this rate to be a weakly decreasing function of productivity.

### 2.2.2 Non-Exporters

Next we describe the conditions that solve the non exporter value function, plus the exit and export thresholds. The problem of non-exporters is described by the following second order ordinary differential equation:

$$(\rho + \delta)V_n(z) = \pi_n z - \kappa_P + \frac{1}{2}V_n(z)^2 z + \frac{1}{2}V_n''(z)(z\sigma)^2$$

We need two border conditions. In addition, we need to determine optimally the exit and export thresholds. Thus, we require a total of four equations. These involve two value matching and two smooth pasting conditions. The value matching conditions determine that, at the exit and export thresholds, the value function of non exporters equals the value of the activity to which the firm is switching, that is, exporting and exiting, respectively. These are:

$$V_n(z_e) = 0$$

$$V_n(z_x) = V_x(z_x)$$

The smooth pasting conditions determine that the slope of the value function must be smooth at the exit and export thresholds.

$$V_n'(z_e) = 0 \tag{19}$$

$$V_n'(z_x) = V_x'(z_x) \tag{20}$$

Appendix B describes why the value matching and smooth pasting conditions should hold at the optimum.

The smooth pasting conditions at the exit and export threshold imply that the growth rate of non-exporters is increasing at some interval. To see this, notice that the smooth pasting conditions, together with the first order conditions for non-exporters, imply:

$$V_n'(z_e) = g_n(z_e) = 0$$

$$V_n'(z_x) = g_n(z_x) = g_x > 0$$

By continuity, it follows that $g_n(z)$ increases for at least some interval. Unfortunately, we cannot rule out non-monotonic behavior.

### 2.3 The Value of the Export Option

To understand the effect of future exports on the value function of non-exporters, it is useful to think of there being an export option that can be exercised only when reaching productivity $z_x$. A firm is willing to pay very little if it is far from $z_x$, since the likelihood of reaching the threshold in the near future is slim. But larger non-exporters would be willing to pay more. The value of this option is already present in the value function $V_n(z)$.

This option value can be computed as the difference between $V_n(z)$ and that of a hypothetical firm that is forced to remain a non-exporter forever. Let $V_h(z)$ be the value of such firm. This can be written as

$$V_h(z) = \max_g \left\{ \pi_n z - \kappa_P - g^2 z + gzV_h'(z) + \frac{1}{2}(z\sigma)^2 V_h''(z) \right\} \tag{21}$$

This hypothetical firm must choose an exit threshold, call this $z_h$. The solution implies

$$V_h(z) = \pi_n z + \frac{1}{2}V_h'(z)^2 z - \kappa_P + \frac{1}{2}(z\sigma)^2 V_h''(z), \quad z \geq z_h$$

To solve this differential equation we need two border conditions. In addition, we need one more equation to find the value $z_h$. The two border conditions are the value matching and smooth pasting at the exit threshold, that is,

$$V_h(z_h) = 0$$

$$V_h'(z_h) = 0$$

We obtain the additional equation for $z_h$ by assuming that a firm with a large enough productivity has a probability of exiting equal to zero. In that case, the firm with such a productivity behaves like an exporter, except that the flow profits of this firm are $\pi_n z$. Then the equation to pin down $z_h$ is given by the growth rate of the hypothetical firm with productivity high enough so that the probability of exiting endogenously becomes zero:

$$\lim_{z\to\infty} V_h'(z) = g_h = (\rho+\delta)\left(1-\sqrt{1-\frac{2\pi_n}{(\rho+\delta)^2}}\right)$$

The value of the export option is, for each $z$, the difference between the value function of a non-exporter $V_n(z)$, and $V_h(z)$.

Section 6 shows quantitatively the value of the export option. It is increasing with productivity, meaning that as the firms come closer to exercising it, they assign a larger value to it. It also increases when trade costs drop.

## 2.4 The Firm Size Distribution

Equation (12) describes the firm size distribution. It is convenient to describe separately the distribution of large firms, where only exporters are present and the mass of entrants is relatively small, from that of smaller firms.

### 2.4.1 Distribution of Large Firms

Given the fact that exporters grow at a constant rate, we can rewrite the Kolmogorov Forward equation for large firms as

$$0 = \mu(z)\left(g_x+\delta-\sigma^2\right) + \mu'(z)z(g_x-2\sigma^2) - \frac{\sigma^2 z^2}{2}\mu''(z) - MG(z)$$

We understand as “large” those productivities where $G(z)$ is close enough to zero. For these firms, the distribution is Pareto, as the following proposition proves.

**Proposition 2** *If $\lim_{z\to\infty} G(z) = 0$, the distribution of large firms approaches a Pareto distribution.*

**Proof:** The Kolmogorov Forward equation for large firms when $G(z) = 0$ is,

$$0 = \mu_L(z)(g_x + \delta - \sigma^2) + \mu_L'(z)z(g_x - 2\sigma^2) - \frac{\sigma^2 z^2}{2}\mu_L''(z)$$

A Pareto distribution solves this equation. To see this, replace $\mu_L(z)$ by a Pareto distribution:

$$\mu_L(z) = Bz^{-b}$$

Inserting this guess into the Kolmogorov Forward equation proves the proposition:

$$0 = g_x + \delta - \sigma^2 - b(g_x - 2\sigma^2) - (-b-1)(-b)\sigma^2/2 \tag{22}$$

■

**Corollary 1** *The parameter $b$ is found by solving equation (22).*

The parameter $B$ is found as a value matching condition, by continuity of the firm size distribution.

### 2.4.2 Distribution of Small Firms

Equation (12) describes the behavior of small firms, that includes non-exporters. This is a second order differential equation, and as such we need two border conditions to determine it. One border condition suggests itself as the measure of firms in the exit threshold should equal zero, that is,

$$\mu(z_e) = 0 \tag{23}$$

The other condition is a smooth pasting condition with the distribution of large firms. Following the continuous behavior of the growth rates for all firms, we impose that the derivative of the firm size distribution be continuous, and this determines the second border condition. In other words, we know that in the limit as $z \to \infty$, $\mu(z) \to Bz^{-b}$, so that $\mu'(z) \to -Bbz^{-b-1}$.

Thus, pick a level of $\hat{z}$ large enough so that $G(\hat{z}) \approx 0$, then the second boundary condition states that at $\hat{z}$, the derivative of the distribution should be continuous. A problem is that $B$ is determined by a value matching condition, so that when determining the bottom part of the distribution $\mu(z)$, $B$ is unknown, and we cannot determine the second border condition necessary to find $\mu(z)$. However, we can compute the elasticity of the distribution at the point $\hat{z}$ as

$$\frac{\mu_L'(\hat{z})}{\mu_L(\hat{z})} = \frac{b}{\hat{z}} \tag{24}$$

Equations (23) and (24) describe the boundary conditions, where $b$ solves equation (22). Finally, $B$ is solved by continuity of the firm size distribution

$$\mu(\hat{z}) = B\hat{z}^{-b}$$

# 3 A Change in Trade Costs

In this section we describe the effects of changing trade costs on firm profits and growth rates under very particular assumptions. These are that $\kappa_P$ is small and that $G(z)$ is bounded above, and the upper bound is less than the export threshold. More precisely, this section operates under the following two assumptions:

A.1 $\kappa_P \to 0$

A.2 $G : [\underline{z}, \bar{z}] \to [0, 1], \bar{z} < z_x$

Assumption A.1 implies that in equilibrium, $z_e \to 0$, independently of trade costs. Assumption A.2 implies that in equilibrium no firm exports instantly after entry.

The main results in this section, all under the assumptions previously described, are that the value of exporters increase with lower trade costs, as well as the value of relatively large non-exporters. The results also apply to growth rates: exporters increase their rate of growth, as well as relatively large non-exporters, when trade costs fall.

Section 6 relaxes these assumptions and explores whether the results in this section extend to a more general case by calibrating the model and simulating a reduction in trade costs.

**Proposition 3** *A drop in trade costs reduces profits for all non-exporters.*

**Proof:** Suppose not, so that a reduction in $\tau$ weakly increases $\pi_n$. Since $\pi_x = (1 + \tau^{1-\nu})\pi_n$, this implies $\pi_x$ increases. Equation (17) shows that $g_x$ also increases, and equation (18) that $V_x(z)$ increases for all $z$. Consider the effect on $V_n(z)$ next. Write the problem of non-exporters as

$$(\rho + \delta)V_n(z) = \max_{g,z_x} \left\{ \pi_n z - \frac{1}{2}g^2 z + zgV_n'(z) + \frac{1}{2}(z\sigma)^2 V_n''(z) - \kappa_P, \quad z \in [z_e, z_x] \right\} \tag{25}$$

$$s.t.$$

$$V_n(z_e) \to_{z_e \to 0} 0, V_n'(z_e) \to_{z_e \to 0} 0, V_n(z_x) = V_x(z_x)$$

Notice that we have not imposed the smooth pasting condition $V_n'(z_x) = V_x'(z_x)$ in the problem above. Instead, we explicitly choose the threshold $z_x$, so that the smooth pasting condition comes up as part of the maximization process. The envelope theorem shows that $V_n(z)$ increases for all $z$:

$$\frac{\partial V_n(z)}{\partial \tau} = \frac{\partial \pi_n}{\partial \tau} z + \lambda_1 \frac{\partial V_x(z)}{\partial \tau} + \lambda_2 \frac{\partial g_x}{\partial \tau}$$

where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ are the Lagrange multipliers. Thus, a reduction in $\tau$ increases $V_n(z)$ for all $z$. But this implies that the free entry condition (11) cannot hold, which is a contradiction. ■

**Proposition 4** *A drop in trade costs increases the value function for the largest non-exporters.*

**Proof:** This follows from the fact that $V_x(z_x)$ must increase when trade costs fall. To see this, notice that Proposition 3 implies that a reduction in $\tau$ reduces $V_n(z)$ for at least some $z$. For the free entry condition to hold, $V_n(z)$ must increase for at least some interval as well. Finally, since $V_n(z)$ is non-decreasing in $z$ because $g_n(z) = V_n'(z) \geq 0$, it follows that $V_n(z_x) = V_x(z_x)$ must increase. ■

This proposition is key to our main results: that at least some non-exporters gain from a reduction in trade costs. Note the importance of the value matching condition. The value matching condition is the way non-exporter incorporate future (potential) export profits into their present value. When the value of an exporter increases, this pushes up the value of a non-exporter via the value matching condition. On the other hand, Proposition 3 states that domestic flow profits fall, which pushes the value of a non-exporter down. As a result, the effect of trade costs on the value of non-exporters is ambiguous, but at least when thresholds are fixed, the value of large non-exporters increases. We explore in section 6 these effects more in detail, allowing for changes in the endogenous thresholds.

**Corollary 2** *A drop in trade costs increases exporter profits.*

**Proof:** Follows from equation (18). ■

**Corollary 3** *A drop in trade costs increases the growth rate of exporters.*

**Proof:** Follows from equation (17). ■

**Corollary 4** *A drop in trade costs increases the growth rate of relatively large non-exporters.*

**Proof:** This is guaranteed by the smooth pasting condition. Continuity implies that this holds for a neighborhood around $z_x$. ■

We were unable to prove the effect of trade costs under more general assumptions that violate A.1 and A.2. To explore this, we calibrate the model and perform counterfactuals. These counterfactuals show that the predictions within this section remain valid.

# 4 Calibration

We calibrate our model to the US economy. Some parameters are set directly according to the data or previous papers. We normalize the number of workers to $L = 1$. We set the elasticity of substitution between varieties $\nu$ as in Rubini (2014) and Ruhl (2008) equal to 2. Rubini (2014) shows that this delivers a proper response of trade volumes to changes in tariffs and Ruhl (2008) shows that this is consistent with high frequency volatility. Moreover, this number is within the estimates of Broda and Weinstein (2006) and close to their median. We set the discount rate so that the steady state risk free interest rate is 4% ($\rho = 0.04$).

We set parameters so that in steady state, 10% of firms die every year. Of these, about 1% are firms with more than 500 employees. Thus, we set $\delta = 0.01$ and calibrate other parameters so that 9% of firms die each year endogenously. This target greatly affects the choice of the fixed production cost $\kappa_P$.

We target a trade volume of 10%, measured as exports over output in the manufacturing industry. We target a share of firms that export of 21%, as in Bernard et al. (2003) when considering only the manufacturing industry. These targets are especially relevant to determine $\tau$ and $\kappa_X$.

The next parameter to calibrate is $\sigma$. This affects the growth rates of non-exporters. In particular, the larger $\sigma$, the lower the drift for the growth rates of non-exporters in equilibrium. Bernard and Jensen (1999) estimate the difference in growth rate between exporters and non-exporters under several specifications. Their findings range from no statistical difference to a difference of 0.4 percentage points in favor of exporters. Accordingly, we target a difference of 0.2 in the rate of growth of exporters relative to non exporters.

Potential entrants draw a productivity parameter from a Normal distribution, with mean $\mu_E$ and variance $\sigma_E$. We set the variance as $\sigma_E = \sigma$, and calibrate $\mu_E$ so that 68.1 percent of the firms that pay the entry cost choose not to produce. This is the exit rate of firms that are less than one year old, as found by Bergoeing et al. (2016).

A last moment that we target is the slope of the distribution of large firms. Axtell (2001) finds that the slope of the firm size distribution for large firms is -1.06. This number is the slope of plotting the log of the proportion of firms with more than $x$ employees against the log of $x$. This moment is particularly relevant to pin down the value of $\kappa_E$.

Table 1 lists all parameters and targeted moments.

Table 1: Parameters and Targets

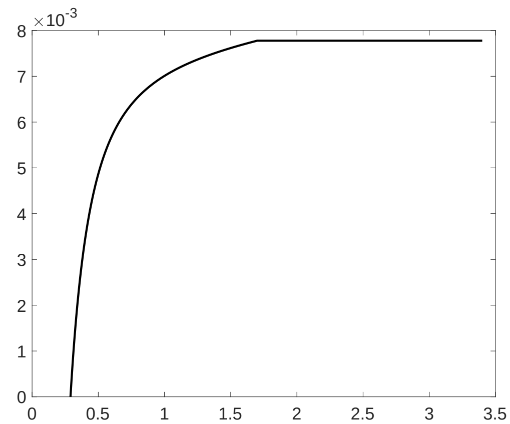
| Parameter | Targeted Moment | Value |
|---|---|---|
| $\rho$ | Risk free interest rate of 4% | 0.04 |
| $\delta$ | Death rate of large firms of 1% | 0.01 |
| $\sigma$ | Growth rate premium of exporters of 0.2 pct. points | 0.24 |
| $\nu$ | High frequency elasticity of trade | 2.00 |
| $\mu_E$ | Exit rate of one year old firms of 68.1% | 0.18 |
| $\tau$ | Exports to Output ratio of 7.5% | 8.30 |
| $\kappa_P$ | Firm death rate of 10% | $2 \times 10^{-4}$ |
| $\kappa_X$ | Share of firms that export of 21% | $6 \times 10^{-6}$ |
| $\kappa_E$ | Slope of firm size distribution for large firms of -1.06 | $2 \times 10^{-4}$ |

# 5 The Calibrated Steady State Equilibrium

This section shows how the calibrated model looks like in equilibrium in steady state. Relatively large firms, with productivity larger than $z_x = 1.70$, export. These grow at a constant rate equal to $g_x = 0.78\%$. Medium sized firms, with productivities smaller than $z_x$ and larger than $z_e = 0.29$, only sell domestically, and their rate of growth is increasing, as Figure 1 shows.

The export option increases both the value of non-exporters, and their growth rates. On average, the present value of non-exporters would be 9.3% lower absent the option to export in the future. This number comes from comparing the actual value of a non-exporter $V_n(z)$, with that of the hypothetical firm that can never export $V_h(z)$, as described in section 2.3, and then taking weighted averages, with the weights determined by the firm size distribution in the new steady state. This average includes numbers very high for low productivities, that would shut

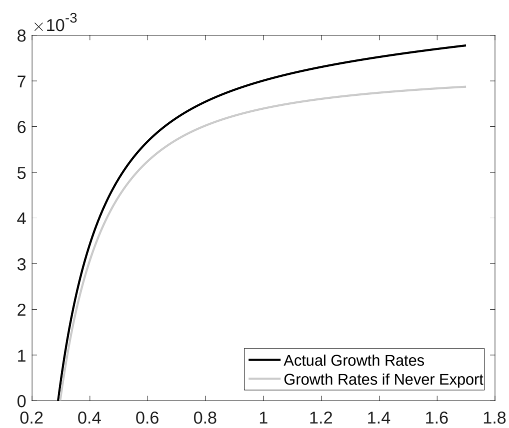
Figure 1: Average firm growth rates across productivities




down absent the export option. For those firms close to the export threshold ($z \approx z_x$), the increase in value from the export option is 9.5%.

The export option also affects growth rates. On average, non-exporters grow at a rate of 0.55%. Of this, 10% (0.05 percentage points) is due to the option value. Figure 2 compares the average growth rates of non-exporters with the average growth rates they would choose absent the possibility of exporting.

Figure 2: Non-exporter growth rates - actual vs. never export

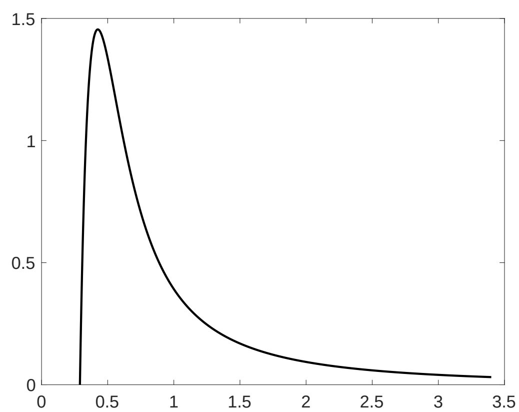



The export option affects all non-exporters, and these effects are heterogeneous across productivities. Among more productive ones, it increases the growth rate by up to 13%. For less productive non-exporters, it prevents the exit of the bottom 0.02% by increasing their present value.

Notice how the export option can help reinterpret a well known fact in the literature. Bernard and Jensen (1999) find that non-exporters increase their growth rate right prior to exporting. They interpret this as evidence that firms export because of exogenous shocks that hit them, making them large enough to find it profitable to export. This is a possible explanation in our model, since a very small firm can always receive an extreme shock, increasing their size and driving them to export. But a more likely outcome is that firms increase their growth prior to exporting because they anticipate the larger profits implied by exporting. Without the export option, the growth rate of firms at the verge of exporting would be 13% lower, showing how important exporting is for firm growth.

Figure 3 shows the firm size distribution in the calibrated steady state. The upper tail of this distribution is Pareto, and it is equal to zero at the exit threshold.

Figure 3: Firm size distribution




# 6 Trade Cost Reduction in the Calibrated Equilibrium

In this section we simulate a reduction in the variable trade costs. We choose as our benchmark a reduction that is large, and $\tau$ by 50%, and report our findings as elasticities, that is, per percentage point reduction in trade costs. We choose a relatively large drop in trade costs to make the changes easier to see graphically, especially on the distribution. This being said, we also experimented with smaller reductions and find very similar elasticities. We compare the equilibria in steady state. We divide our results into the effects on firm value, growth rates, and other results in the subsections below.

## 6.1 Firm Value

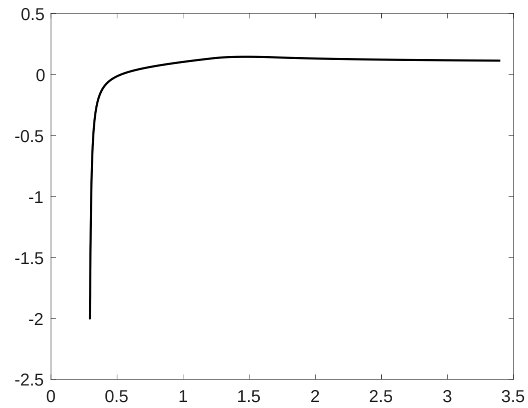
A reduction in trade costs increases the value for all exporters. On average, the elasticity of the value of exporters to trade costs is -0.13%, so that for each percentage point drop in trade costs, an exporter on average increases its value by 0.13%. This average includes firms that were not exporting before the change in trade costs, and gain relatively more, and firms that export before and after the drop. New exporters gain 0.14%, and old exporters gain 0.126%.

Among non-exporters, the effects are heterogeneous. Relatively large ones gain from the drop in trade costs, since these firms discount the possibility of exporting using a relatively small factor, while small ones discount it using a larger factor, and therefore the value of the option to export is small. On average, non-exporters lose value. A one percent drop in trade costs reduces the value of non-exporters by 0.01% on average. But this number includes some firms that shut down, losing 100% of their value, and some firms that actually gain. This is because, as in Melitz (2003), the exit threshold increases, so more firms exit. In all, 59% of non-exporters are better off with the lower trade costs.[4] This share reveals that most non-exporters gain from the reduction in tariffs, a remarkable difference with mainstream models of international trade that predict that these firms are worse off. This share barely changes with different changes in trade costs.

Figure 4 illustrates the gain across different types of firms. A value of zero implies that firm value stays constant. Clearly, it is small firms that lose, but by no means non-exporters, since firms are exporters when $z > 1.22$, and all firms with $z > 0.53$ gain from the drop in trade costs.

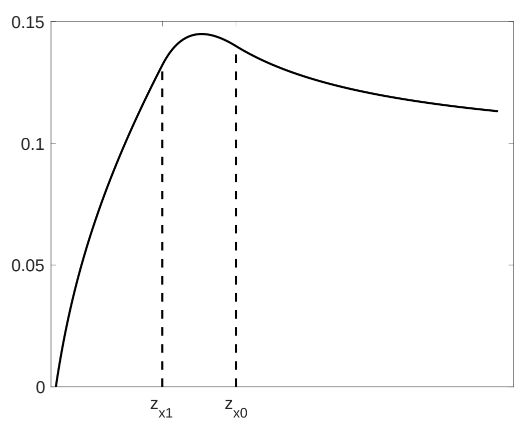
[4] This number does not include firms that became exporters after the drop in trade costs.

Figure 4: Negative of Elasticity of Firm Value to Trade Costs




A problem with Figure 4 is that the large losses by firms that are close to the export threshold distort the plot, so it is hard to see the pattern of firm gains. Figure 5 drops firms that lose in value. From this, it is clear that the firms that increase in value the most are new exporters. What is also clear, is that many non-exporters that are close to the export threshold gain more than some exporters.

Figure 5: Negative of Elasticity of Firm Value to Trade Costs - Firms that Gain



## 6.2 Growth Rates

Figure 6 illustrates the change in growth rates. The figure shows that exporters and large non-exporters grow faster. The behavior of small exporters is harder to see. Figure 7 zooms in into small firms, to show that small firms reduce their growth rates. In fact, all firms with $z > 0.41$ increase their growth rates. This implies that some firms lose value and increase their growth rates (firms with $z \in (0.41, 0.53)$).

Figure 6: Firm growth rates under different trade costs

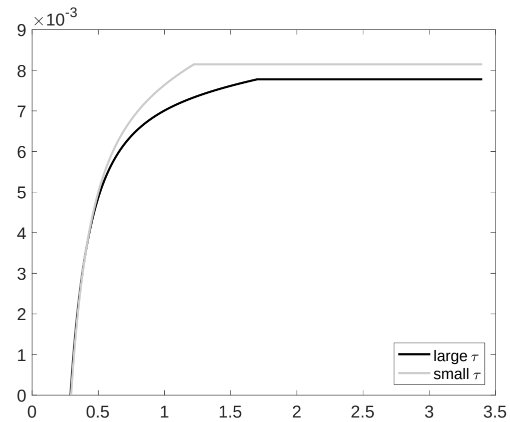
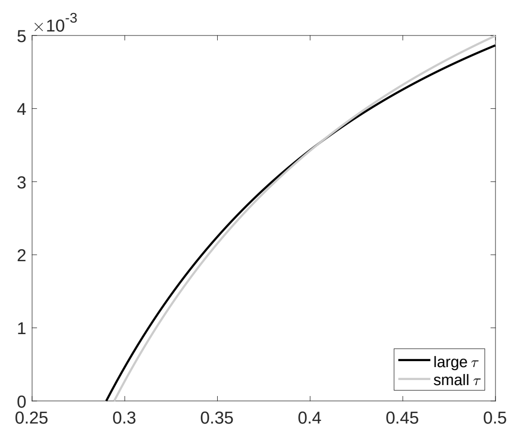



Figure 7: Growth rates under different trade costs (small non-exporters)




Exporter growth rates increase because their flow profits increase. A one percentage point drop in trade costs increases exporter profits by 0.09%, the same as exporter growth rates.

The reaction of non-exporters is heterogeneous. Large non-exporters grow faster, and small ones grow slower. The reason lies in the fact that there are two channels affecting the growth rates of non-exporters. On one hand, flow profits fall. Quantitatively, by 0.12% per percentage point drop in trade costs. This pushes growth rates down. On the other hand, the export option increases in value, which pushes growth rates up. The increase is larger for larger non-exporters, as we show in Figure 8. In the aggregate, the second channel dominates, increasing the average growth of non-exporters by 0.02%, with the increase being larger for larger firms.

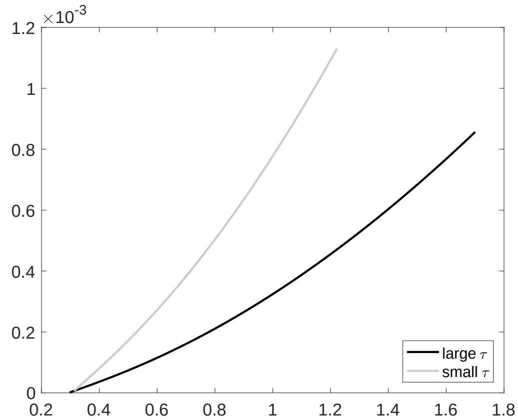
Innovation rates also increase. The elasticity of the ratio of innovation to sales is -0.12 across all firms. When separately analyzing exporters and non-exporters, the elasticity is larger for non-exporters, driven in part by the very low ratios in very small firms (recall that firms close to the exit threshold do not innovate). For non-exporters, the average elasticity is -0.14, and for exporters it is -0.10.

These findings are consistent with Bustos (2011).[5] She finds that firms in all quartiles of the distribution increase innovation when trade costs drop between Argentina and Brazil. In particular, product and process innovation increase more for firms in the second quartile than those in the first quartile, consistent with our model.[6] Eslava et al. (2015) also finds that non-exporters increase their innovation rates when trade costs fall. Their proxy for innovation is the skill intensity within a firm. Along these lines, Topalova and Khandelwal (2011) find that lower trade costs increase the productivity of small firms in India. While they do not discern between exporters and non-exporters, it is likely that within small firms, a great number of them are not exporters. Finally, Newman et al. (2013) also find that non-exporters in Vietnam increase innovation after trade costs drop.

[5]See Table 7.

[6]Although some of these results are not statistically significant.

Figure 8: Present value of the export option under different trade costs




## 6.3 Other Effects

This section describes the changes observed in other areas not directly related to firm value and growth rates. As in Melitz (2003) and others, a drop in trade costs reduces the export threshold, driving more firms into the export market, and increases the exit threshold, increasing exit and the exit rate.

Figure 9 shows the change in the firm size distribution. The distribution moves to the right, so that the average productivity increases. Figure 10 zooms in on large firms, and Figure 11 on small ones to make the changes more evident. The elasticity of the average firm productivity with respect to trade costs is -0.34, where average firm productivity is defined as

$$av.prod = \frac{\int_{z_e}^{\infty} z^{\frac{1}{\nu-1}} \mu(z)dz}{\int_{z_e}^{\infty} \mu(z)dz}$$

Entry drops with trade costs. The elasticity of entry with respect to trade costs is 0.34. This is detrimental to welfare. However, welfare increases across steady states, with an elasticity of -0.20. Table 2 summarizes many of the statistics described in this section.

Figure 9: The firm size distribution under different trade costs

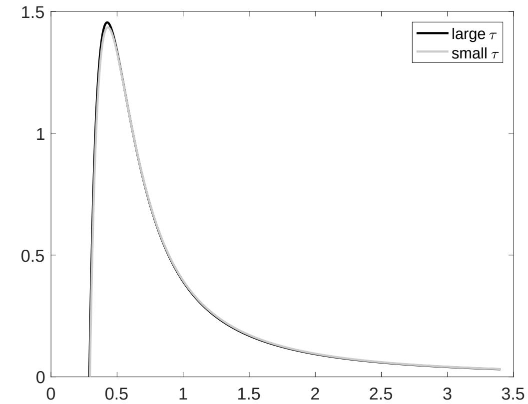
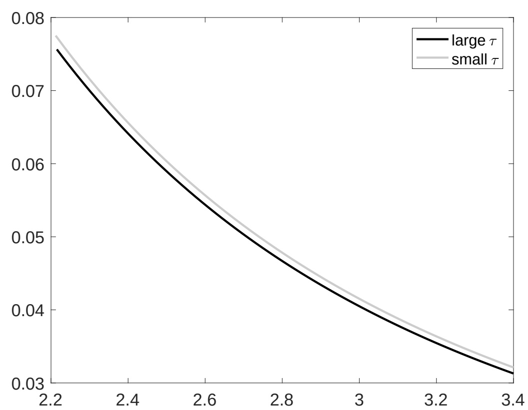



Figure 10: The firm size distribution under different trade costs (large firms)

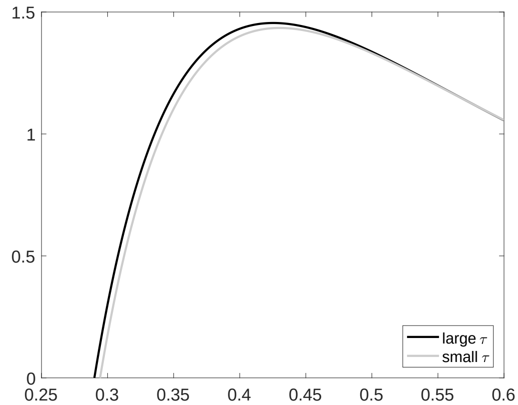



# 7 Conclusion

We develop a tractable model of international trade where firms are heterogeneous and growth decisions are endogenous. Our model delivers features that are empirically consistent: exporters grow faster than non-exporters, non-exporters increase their growth rates prior to becoming non-exporters, and non-exporters increase their innovation expenses following a trade liberalization.

This challenges the well established belief that lower trade costs only benefit exporters, and non-exporters suffer at their expense. On the contrary, we find that most non-exporters gain from trade liberalizations from the present value of future exports. Moreover, lower trade costs provide incentives for most of these firms to increase innovation and growth rates. It is only extremely

Figure 11: The firm size distribution under different trade costs (small firms




small non-exporters that lose.

Our findings can account for the observation in several empirical papers mentioned previously that during trade liberalizations, both exporters and non-exporters are observed increasing their levels of innovation. And to do that, we do not need to resort to non-standard stories about input and output linkages between firms that are purely domestic, and firms that export. This does not imply that these mechanisms do not exist. In fact, we see our paper as complementing those mechanisms previously proposed.

We also suggest that a reason why non-exporters increase their growth rate prior to exporting is driven by the incentives to export. This challenges the interpretation that firms grow faster for reasons unrelated to trade, and being large drives them to start exporting. In fact, the option to export adds 10% to the growth rates of the average non-exporter, and 13% to the largest ones.

We see our paper as uncovering one additional gain to openness. Not only do exporters gain in value and efficiency. Non-exporters gain as well. Policy makers should abandon the idea that trade liberalizations only help exporting firms. While non-exporting flow profits may drop, their present value in most cases increases with liberalization. In fact, trade liberalizations improves overall efficiency and better aligns incentives in a dynamic sense, both for exporters and non-exporters. It is only the most inefficient non-exporters that lose.

Table 2: The elasticity of equilibrium moments with respect to trade costs

| Elasticity with respect to trade costs | |
|---|---|
| $\pi_n$ | 0.12 |
| $\pi_x$ | -0.09 |
| Export threshold | 0.56 |
| Exit threshold | $-0.03$ |
| Exporter avg. growth rate | -0.09 |
| Non-exporter avg. growth rate | -0.02 |
| Large Non-exporter avg. growth rate | -0.12 |
| Small Non-exporter avg. growth rate | 0.55 |
| Innovation to sales ratio | -0.12 |
| Exporter innovation to sales ratio | -0.10 |
| Non-exporter innovation to sales ratio | -0.14 |
| Average productivity | -0.34 |
| Entry | 0.34 |
| Welfare | $-0.20$ |

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# A No Constant Growth Equilibrium when $2\pi_x > (\rho+\delta)^2$

We show that there is no equilibrium when $2\pi_x > (\rho+\delta)^2$ in a discretized model, with no uncertainty on firm growth. The intuition of this proof carries through to the case with uncertainty as well.

For simplicity, let the time interval $dt = 1$. Also, let $b = \frac{1}{1+\rho+\delta}$. Consider the problem of an exporter

$$V_x(z) = \max_{g_t} \left(\pi_x - \frac{g_t^2}{2}\right) z_t + \beta V_x(z_{t+1})$$

$$s.t. \;\; z_{t+1} = (1+g_t)z_t$$

The first order conditions set

$$g_t = \beta V_x'((1+g_t)z_t)$$

Using the envelope theorem:

$$V_x'(z_t) = \left(\pi_x - \frac{g_t^2}{2}\right) + \beta V_x'(z_{t+1})(1+g_t)$$

Adding the first order condition above,

$$V_x'(z_t) = \left(\pi_x - \frac{g_t^2}{2}\right) + g_t(1+g_t)$$

Therefore, the growth rates for exporters satisfy the following law of motion:

$$g_t = \beta\left[\left(\pi_x - \frac{g_{t+1}^2}{2}\right) + g_{t+1}(1+g_{t+1})\right]$$

Hence,

$$\frac{g_t - g_{t+1}}{\beta} = -(\rho+\delta)g_{t+1} + \pi + \frac{g_{t+1}^2}{2} = 0 \tag{26}$$

The solution to this equation sets $g = (1 - \sqrt{(}1 - 2\pi_x/(\rho+\delta)^2))$, which is an imaginary number. This means that the system never sets $g_t = g_{t+1}$. In fact, given that the profits of large firms should be non-negative, it turns out that $-(\rho+\delta)g_{t+1} + \pi + \frac{g_{t+1}^2}{2} > 0$, so $g_t > g_{t+1}$. What is more, setting $g_{t+1} = \rho+\delta$ minimizes the right hand side of equation (26), which determines that

$$g_t - g_{t+1} \geq \beta\left(\pi_x - \frac{(\rho+\delta)^2}{2}\right) > 0$$

Thus, the difference between $g_t$ and $g_{t+1}$ is strictly larger than zero for all $z$. Since $\lim_{t\to\infty} g_g \geq 0$, this implies that $\lim_{t\to 0} g_t \to \infty$, which is not feasible.

# B Value Matching and Smooth Pasting

To understand the value matching and smooth pasting conditions, consider first the value matching conditions. These conditions are set such that firms are better off exporting only when $z > z_x$, they are better off not exporting when $z \in (z_e, z_x)$, and they are better off shutting down when $z < z_e$. That is,

$$0 \geq \max\{V_n(z), V_x(z)\}, \quad \forall z \leq z_e, \tag{27}$$

$$V_n(z) \geq \max\{0, V_x(z)\}, \quad \forall z \in [z_e, z_x], \tag{28}$$

$$V_x(z) \geq \max\{0, V_n(z)\}, \quad \forall z \geq z_x \tag{29}$$

Continuity and monotonicity imply the value matching conditions. Continuity is easy to verify for both $V_n(z)$ and $V_x(z)$. Monotonicity comes from the fact $V_i'(z) = g_i(z) \geq 0, i = n, x$.

Next consider the smooth pasting condition at the export threshold. If this did not hold, then we show that the firm would not be maximizing profits. If $V_n'(z_x) > V_x'(z_x)$ and $V_n(z_x) = V_x(z_x)$, then there exists some $\epsilon > 0$ such that $V_n(z_x + \epsilon) > V_x(z_x + \epsilon)$, violating condition (29). If, on the other hand, $V_n'(z_x) < V_x'(z_x)$, then we show that there exists some $z_{x2} \neq z_x$ such that starting to export at $z_{x2}$ yields higher profits. To show this, we develop the intuition using a discrete approximation to the problem. Let $\Delta t$ be the time interval, and $\Delta h$ the step size in the productivity space. For a given small $\Delta t$, let $\Delta h = \sigma\sqrt{\Delta t}$. Consider the alternative policy of a non-exporter that instead of becoming an exporter when it reaches $z_x$, it waits for an additional period. If productivity increases, it switches to exporting, and if it decreases, it remains as a non-exporter. Let $p = 1/2(1 + g(z)z/\sigma^2\Delta h)$ be the probability of having a step up, and $q = 1/2(1 - g(z)z/\sigma^2\Delta h)$ be the probability of a step down.[7] Let the value of this alternative policy be $\tilde{V}_n(z_x)$:

$$\begin{aligned} \tilde{V}_n(z_x) &= (\pi_x z_x - g(z)^2 z - \kappa_P)\Delta t + (1 - (\rho + \delta)\Delta t)[pV_x(z_x + \Delta h) + qV_n(z_x - \Delta h)] = \\ &= (\pi_x z_x - g(z)^2 z - \kappa_P)\Delta t + (1 - (\rho + \delta)\Delta t)[V_n(z_x) + (pV_x'(z_x) - qV_n'(z_x))\Delta h + \dots] \end{aligned} \tag{30}$$

where "…" includes terms with $\Delta h$ elevated to a power larger than or equal to 2, and $g(z)$ is the optimal growth rate.

Notice that $V_n(z) = (\pi_x z_x - g(z)^2 z - \kappa_P)\Delta t + (1 - (\rho + \delta)\Delta t)V_n(z)$. To see this,

$$\begin{aligned} V_n(z) &= (\pi_x z_x - g(z)^2 z - \kappa_P)\Delta t + (1 - (\rho + \delta)\Delta t)[pV_n(z + \Delta h) + qV_n(z - \Delta h) \\ &= (\pi_x z_x - g(z)^2 z - \kappa_P)\Delta t + (1 - (\rho + \delta)\Delta t)[V_n(z) + V_n'(z)\Delta h(p - q)] \\ &= (\pi_x z_x - g(z)^2 z - \kappa_P)\Delta t + (1 - (\rho + \delta)\Delta t)V_n(z) \end{aligned}$$

where the last equality comes from ignoring terms with $\Delta h$ elevated to a power larger than or equal to 2, since $p - q = g(z)z/\sigma^2\Delta h$.

[7] This intuition is based on Dixit (1993), including the relationship between $\Delta h, \Delta t, p$ and $q$. The book also contains a more formal proof of the smooth pasting conditions.

Inserting this into equation (30), and using the value matching condition,

$$\tilde{V}_n(z_x) = V_x(z_x) + \frac{1}{2}\Delta h(V_x'(z_x) - V_n'(z_x)) > V_x(z_x) \tag{31}$$

which suggests that the alternative policy of waiting one more period has a higher expected value than switching immediately upon reaching $z_x$.